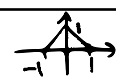
Fourier Transforms and Relations (Chap. 23)

	Discrete Fourier Transform $\mathcal{F}_D$	Fourier Series $\mathcal{F}_T$	Discrete Time Fourier Transform $\mathcal{F}_Z$	Fourier Transform $\mathcal{F}_R$
$\mathcal{T}$	$\mathbb{Z}_n$	$\mathbb{R}_T$	$\mathbb{Z}$	$\mathbb{R}$
$\mathcal{F}$	$\{\frac{\nu}{n} : \nu \in \mathbb{Z}_n\}$	$\{\frac{\nu}{T} : \nu \in \mathbb{Z}\}$	$[-\frac{1}{2}, \frac{1}{2})$	$\mathbb{R}$
$X(f)$	$\sum_{t=0}^{n-1} x(t) \frac{e^{-i2\pi ft}}{\sqrt{n}}$	$\int_0^T x(t) \frac{e^{-i2\pi ft}}{\sqrt{T}} dt$	$\sum_{t=-\infty}^{+\infty} x(t) e^{-i2\pi ft}$	$\int_{-\infty}^{+\infty} x(t) e^{-i2\pi ft} dt$
$x(t)$	$\sum_{f \in \mathcal{F}} X(f) \frac{e^{i2\pi ft}}{\sqrt{n}}$	$\sum_{f \in \mathcal{F}} X(f) \frac{e^{i2\pi ft}}{\sqrt{T}}$	$\int_{-\frac{1}{2}}^{\frac{1}{2}} X(f) e^{i2\pi ft} df$	$\int_{-\infty}^{+\infty} X(f) e^{i2\pi ft} df$

Fourier Transform Properties

FT Property	Time Domain	$\longleftrightarrow$	Frequency Domain
Convolution	$x(t) * y(t)$	$\longleftrightarrow$	$X(f)Y(f)$
Linearity	$\alpha x(t) + \beta y(t)$	$\longleftrightarrow$	$\alpha X(f) + \beta Y(f)$
Time Shift	$x(t - t_0)$	$\longleftrightarrow$	$e^{-i2\pi f t_0} X(f)$
Frequency Shift	$e^{i2\pi f_0 t} x(t)$	$\longleftrightarrow$	$X(f - f_0)$
Inner Product	$\int_{-\infty}^{\infty} x(t) y^*(t) dt$	$=$	$\int_{-\infty}^{\infty} X(f) Y^*(f) df$
Parseval's Theorem	$\int_{-\infty}^{\infty} \|x(t)\|^2 dt$	$=$	$\int_{-\infty}^{\infty} \|X(f)\|^2 df$
Spectral Conjugation	$x^*(-t)$	$\longleftrightarrow$	$X^*(f)$
Autocorrelation	$x(t) * x^*(-t) = \int_{-\infty}^{\infty} x(\lambda) x^*(\lambda - t) d\lambda$	$\longleftrightarrow$	$\|X(f)\|^2$
Modulation	$x(t)y(t)$	$\longleftrightarrow$	$X(f) * Y(f)$
Scaling	$x(ct)\ c \neq 0$	$\longleftrightarrow$	$\frac{1}{\|c\|} X\left(\frac{f}{c}\right)$
Differentiation	$dx(t)/dt$	$\longleftrightarrow$	$(i2\pi f) X(f)$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda$	$\longleftrightarrow$	$\frac{X(f)}{(i2\pi f)} + \frac{1}{2} X(0) \delta_D(f)$
Real ~ HS	$\text{Re}\{x(t)\}$	$\longleftrightarrow$	$\mathbb{HS}\{X(f)\}$
Imaginary ~ HAS	$\text{Im}\{x(t)\}$	$\longleftrightarrow$	$-i\mathbb{HAS}\{X(f)\}$
HS ~ Real	$\mathbb{HS}\{x(t)\}$	$\longleftrightarrow$	$\text{Re}\{X(f)\}$
HAS ~ Imaginary	$\mathbb{HAS}\{x(t)\}$	$\longleftrightarrow$	$i\text{Im}\{X(f)\}$
Duality	$X(t)$	$\longleftrightarrow$	$x(-f)$

Fourier Transform Pairs

No.	**Signal** $:x(t)$	**Transform** $:X(f)$
⟨1⟩	1	$\delta_D(f)$
⟨2⟩	$\delta_D(t-t_0)$	$e^{-\mathrm{i}2\pi f t_0}$
⟨3⟩	$e^{\mathrm{i}2\pi f_0 t}$	$\delta_D(f-f_0)$
⟨4⟩	$\sum_{n=-\infty}^{\infty} \delta_D(t-nT_s)$	$\frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta_D(f-n/T_s)$
⟨5⟩	$\cos(2\pi f_0 t)$	$\frac{1}{2}(\delta_D(f-f_0)+\delta_D(f+f_0))$
⟨6⟩	$\sin(2\pi f_0 t)$	$\frac{1}{2\mathrm{i}}(\delta_D(f-f_0)-\delta_D(f+f_0))$
⟨7⟩	$\mathrm{sgn}(t)$	$1/(\pi f)$
⟨8⟩	$u(t)$	$1/(2\pi f)+\frac{1}{2}\delta_D(f)$
⟨9⟩	$\mathrm{rect}\,(t)$ 1 (-1≤t≤1)	$\mathrm{sinc}\,(f)$
⟨10⟩	$\mathrm{trian}\,(t)$ 	$(\mathrm{sinc}\,(f))^2$
⟨11⟩	$e^{-\pi t^2}$	$e^{-\pi f^2}$
⟨12⟩	$e^{-a\lvert t\rvert}$ $a>0$ $(real)$	$\frac{2a}{(2\pi f)^2+a^2}$
⟨13⟩	$e^{-at}u(t)$, $\mathrm{Re}\,\{a\}>0$	$\frac{1}{\mathrm{i}2\pi f+a}$
⟨14⟩	$\frac{1}{a-b}\left(e^{-bt}-e^{-at}\right)u(t)$, $\mathrm{Re}\,\{a\}$, $\mathrm{Re}\,\{b\}>0$ $a\neq b$	$\frac{1}{(\mathrm{i}2\pi f+a)(\mathrm{i}2\pi f+b)}$
⟨15⟩	$e^{-at}\sin(2\pi f_0 t)u(t)$, $\mathrm{Re}\,\{a\}>0$	$\frac{2\pi f_0}{[\mathrm{i}2\pi f-(a+\mathrm{i}2\pi f_0)][\mathrm{i}2\pi f-(a-\mathrm{i}2\pi f_0)]}$
⟨16⟩	$te^{-at}u(t)$, $\mathrm{Re}\,\{a\}>0$	$\frac{1}{(\mathrm{i}2\pi f+a)^2}$
⟨17⟩	$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\mathrm{Re}\,\{a\}>0$	$\frac{1}{(\mathrm{i}2\pi f+a)^n}$

Discrete Time Fourier Transform Properties

DTFT Property	Time Domain	$\longleftrightarrow$	Periodic $\langle 1 \rangle$ Frequency Domain
Convolution	$x_n * y_n$	$\longleftrightarrow$	$X(\nu)Y(\nu)$
Linearity	$\alpha x_n + \beta y_n$	$\longleftrightarrow$	$\alpha X(\nu) + \beta Y(\nu)$
Time Shift	x_{n-n_0}	$\longleftrightarrow$	$e^{-\mathrm{i}2\pi\nu n_0} X(\nu)$
Frequency Shift	$e^{\mathrm{i}2\pi\nu_0 n} x_n$	$\longleftrightarrow$	$X(\nu - \nu_0)$
Inner Product	$\sum_{n=-\infty}^{\infty} x_n y_n^*$	$=$	$\int_0^1 X(\nu)Y^*(\nu)d\nu$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$=$	$\int_0^1 \lvert X(\nu)\rvert^2 d\nu$
Spectral Conjugation	x_{-n}^*	$\longleftrightarrow$	$X^*(\nu)$
Autocorrelation	$x_n * x_{-n}^*$	$\longleftrightarrow$	$\lvert X(\nu)\rvert^2$
Modulation	$x_n y_n$	$\longleftrightarrow$	$X(\nu) \circledast_1 Y(\nu)$
Scaling	$x_{rn} r \in \mathcal{Z}\ r \neq 0$	$\longleftrightarrow$	$\frac{1}{\lvert r\rvert} \sum_{m=0}^{r-1} X\left(\frac{\nu+m}{r}\right)$
Time Difference	$x_n - x_{n-1}$	$\longleftrightarrow$	$(1 - e^{-\mathrm{i}2\pi\nu})X(\nu)$
Summation	$\sum_{m=-\infty}^{n} x_m$	$\longleftrightarrow$	$(1 - e^{-\mathrm{i}2\pi\nu})^{-1} X(\nu) + \frac{1}{2} X(0) \sum_{r=-\infty}^{\infty} \delta_D(\nu - r)$
Real $\sim$ HS	$\mathrm{R}\textsc{e}\{x_n\}$	$\longleftrightarrow$	$\mathbb{HS}\{X(\nu)\}$
Imaginary $\sim$ HAS	$\mathrm{I}\textsc{m}\{x_n\}$	$\longleftrightarrow$	$-\mathrm{i}\mathbb{HAS}\{X(\nu)\}$
HS $\sim$ Real	$\mathbb{HS}\{x_n\}$	$\longleftrightarrow$	$\mathrm{R}\textsc{e}\{X(\nu)\}$
HAS $\sim$ Imaginary	$\mathbb{HAS}\{x_n\}$	$\longleftrightarrow$	$\mathrm{i}\mathrm{I}\textsc{m}\{X(\nu)\}$
Duality with FS	$\mathbb{FS}\{X(t/T)\}$	$=$	x_{-k}

Discrete Time Fourier Transform Pairs

No.	**Signal:** x_n	**Periodic** $\langle 1 \rangle$ **Transform:** $X(\nu)$
$\langle 1 \rangle$	1	$\sum_{m=-\infty}^{\infty} \delta_D(\nu - m)$
$\langle 2 \rangle$	$\delta_K[n - n_0]$	$e^{-\mathrm{i}2\pi\nu n_0}$
$\langle 3 \rangle$	$e^{\mathrm{i}2\pi\nu_0 n}$	$\sum_{m=-\infty}^{\infty} \delta_D(\nu - \nu_0 - m)$
$\langle 4 \rangle$	$\sum_{m=-\infty}^{\infty} \delta_K[n - mM]$	$\frac{1}{M} \sum_{m=-\infty}^{\infty} \delta_D(\nu - m/M)$
$\langle 5 \rangle$	$\cos(2\pi\nu_0 n)$	$\frac{1}{2} \sum_{m=-\infty}^{\infty} (\delta_D(\nu - \nu_0 - m) + \delta_D(\nu + \nu_0 - m))$
$\langle 6 \rangle$	$\sin(2\pi\nu_0 n)$	$\frac{1}{2\mathrm{i}} \sum_{m=-\infty}^{\infty} (\delta_D(\nu - \nu_0 - m) - \delta_D(\nu + \nu_0 - m))$
$\langle 7 \rangle$	$\mathrm{sgn}\,[n]$	$\frac{2}{1-e^{-\mathrm{i}2\pi\nu}}$
$\langle 8 \rangle$	$\mathrm{u}\,[n]$	$\frac{1}{1-e^{-\mathrm{i}2\pi\nu}} + \frac{1}{2} \sum_{m=-\infty}^{\infty} \delta_D(\nu - m)$
$\langle 9 \rangle$	$\mathrm{Drect}_M\,[n]$	$(2M+1)\mathrm{dinc}_M\,(\nu) = \frac{\sin(2\pi\nu(M+1/2))}{\sin(\pi\nu)}$
$\langle 10 \rangle$	$\mathrm{Dtrian}_M\,[n]$	$((2M+1)\mathrm{dinc}_M\,(\nu))^2 = \left(\frac{\sin(2\pi\nu(M+1/2))}{\sin(\pi\nu)}\right)^2$
$\langle 11 \rangle$	$2V\mathrm{sinc}\,(2Vn) \quad 0 < V < 1/2$	$\sum_{m=-\infty}^{\infty} \mathrm{rect}\left(\frac{\nu - m}{2V}\right)$
$\langle 12 \rangle$	$\rho^{\|n\|} \quad \|\rho\| < 1$ (real)	$\frac{1-\rho^2}{1+\rho^2-2\rho\cos(2\pi\nu)}$
$\langle 13 \rangle$	$a^n\mathrm{u}\,[n] \quad \|a\| < 1$	$\frac{1}{1-ae^{-\mathrm{i}2\pi\nu}}$
$\langle 14 \rangle$	$\frac{1}{a-b}\left(b^{n+1} - a^{n+1}\right)\mathrm{u}\,[n] \quad \|a\|,\ \|b\| < 1\ a \neq b$	$\frac{1}{(1-ae^{-\mathrm{i}2\pi\nu})(1-be^{-\mathrm{i}2\pi\nu})}$
$\langle 15 \rangle$	$\rho^n \sin[2\pi\nu_0(n+1)]\mathrm{u}\,[n] \quad \|\rho\| < 1$ (real)	$\frac{\sin(2\pi\nu_0)}{(1-\rho e^{\mathrm{i}2\pi\nu_0}e^{-\mathrm{i}2\pi\nu})(1-\rho e^{-\mathrm{i}2\pi\nu_0}e^{-\mathrm{i}2\pi\nu})}$
$\langle 16 \rangle$	$(n+1)a^n\mathrm{u}\,[n] \quad \|a\| < 1$	$\frac{1}{(1-ae^{-\mathrm{i}2\pi\nu})^2}$
$\langle 17 \rangle$	$\frac{(n+r-1)!}{n!(r-1)!}a^n\mathrm{u}\,[n] \quad \|a\| < 1$	$\frac{1}{(1-ae^{-\mathrm{i}2\pi\nu})^r}$

Fourier Series Properties

FS Property	Periodic (T) Time Domain	$\longleftrightarrow$	Frequency Domain
Convolution	$x(t) \circledast_T y(t)$	$\longleftrightarrow$	$T X_k Y_k$
Linearity	$\alpha x(t) + \beta y(t)$	$\longleftrightarrow$	$\alpha X_k + \beta Y_k$
Time Shift	$x(t - t_0)$	$\longleftrightarrow$	$e^{-\mathrm{i}\frac{2\pi}{T}kt_0} X_k$
Frequency Shift	$e^{\mathrm{i}\frac{2\pi}{T}k_0 t} x(t)$	$\longleftrightarrow$	X_{k-k_0}
Inner Product	$\frac{1}{T}\int_0^T x(t) y^*(t) dt$	$=$	$\sum_{k=-\infty}^{\infty} X_k Y_k^*$
Parseval's Theorem	$\frac{1}{T}\int_0^T \lvert x(t)\rvert^2 dt$	$=$	$\sum_{k=-\infty}^{\infty} \lvert X_k\rvert^2$
Spectral Conjugation	$x^*(-t)$	$\longleftrightarrow$	X_k^*
Autocorrelation	$\frac{1}{T} x(t) \circledast_T x^*(-t)$	$\longleftrightarrow$	$\lvert X_k\rvert^2$
Modulation	$x(t)y(t)$	$\longleftrightarrow$	$X_k * Y_k$
Scaling	$x(rt) \;\; r \in \mathcal{Z} \; r \neq 0$	$\longleftrightarrow$	$\mathrm{sgn}\,[r] \sum_{m=-\infty}^{\infty} X_m \mathrm{sinc}\,(k - mr)$
Differentiation	$dx(t)/dt$	$\longleftrightarrow$	$\left(\mathrm{i}\frac{2\pi}{T}k\right) X_k$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda$	$\longleftrightarrow$	$\left(\mathrm{i}\frac{2\pi}{T}k\right)^{-1} X_k$ if $X_0 = 0$
Real $\sim$ HS	$\mathrm{Re}\,\{x(t)\}$	$\longleftrightarrow$	$\mathbb{HS}\,\{X_k\}$
Imaginary $\sim$ HAS	$\mathrm{Im}\,\{x(t)\}$	$\longleftrightarrow$	$-\mathrm{i}\mathbb{HAS}\,\{X_k\}$
HS $\sim$ Real	$\mathbb{HS}\,\{x(t)\}$	$\longleftrightarrow$	$\mathrm{Re}\,\{X_k\}$
HAS $\sim$ Imaginary	$\mathbb{HAS}\,\{x(t)\}$	$\longleftrightarrow$	$\mathrm{i}\mathrm{Im}\,\{X_k\}$
Duality with DTFT	$\mathbb{DTFT}\,\{X_n\}$	$=$	$x(-\nu T)$

Fourier Series Pairs

No.	**Periodic (T) Signal:** $x(t)$	**Transform:** X_k
$\langle 1 \rangle$	1	$\delta_K[k]$
$\langle 2 \rangle$	$e^{\mathrm{i}\frac{2\pi}{T}k_0 t}$	$\delta_K[k-k_0]$
$\langle 3 \rangle$	$\sum_{m=-\infty}^{\infty} \delta_D(t-mT)$	$\frac{1}{T}\ \forall\, k$
$\langle 4 \rangle$	$\cos\left(\frac{2\pi}{T}k_0 t\right)$	$\frac{1}{2}(\delta_K[k-k_0]+\delta_K[k+k_0])$
$\langle 5 \rangle$	$\sin\left(\frac{2\pi}{T}k_0 t\right)$	$\frac{1}{2\mathrm{i}}(\delta_K[k-k_0]-\delta_K[k+k_0])$
$\langle 6 \rangle$	$\sum_{m=-\infty}^{\infty} \mathrm{rect}\left(\frac{t-mT}{T_0}\right) \quad 0<T_0<T$	$\frac{T_0}{T}\mathrm{sinc}\left(\frac{kT_0}{T}\right)$
$\langle 7 \rangle$	$\sum_{m=-\infty}^{\infty} \mathrm{trian}\left(\frac{t-mT}{T_0}\right) \quad 0<T_0<T/2$	$\frac{T_0}{T}\left(\mathrm{sinc}\left(\frac{kT_0}{T}\right)\right)^2$

Discrete Fourier Transform Properties

DFT Property	Time Domain (mod N)	$\longleftrightarrow$	Frequency Domain (mod N)
Convolution	$x[n] \circledast_N y[n]$	$\longleftrightarrow$	$X[k]Y[k]$
Linearity	$\alpha x[n] + \beta y[n]$	$\longleftrightarrow$	$\alpha X[k] + \beta Y[k]$
Time Shift	$x[n-n_0]$	$\longleftrightarrow$	$e^{-\mathrm{i}\frac{2\pi}{N}kn_0} X[k]$
Frequency Shift	$e^{\mathrm{i}\frac{2\pi}{N}k_0 n} x[n]$	$\longleftrightarrow$	$X[k-k_0]$
Inner Product	$\sum_{n=0}^{N-1} x[n]y^*[n]$	$=$	$\frac{1}{N}\sum_{k=0}^{N-1} X[k]Y[k]^*$
Parseval's Theorem	$\sum_{n=0}^{N-1} \lvert x[n]\rvert^2$	$=$	$\frac{1}{N}\sum_{k=0}^{N-1} \lvert X[k]\rvert^2$
Spectral Conjugation	$x^*[-n]$	$\longleftrightarrow$	$X^*[k]$
Autocorrelation	$x[n] \circledast_N x^*[-n]$	$\longleftrightarrow$	$\lvert X[k]\rvert^2$
Modulation	$x[n]y[n]$	$\longleftrightarrow$	$\frac{1}{N} X[k] \circledast_N Y[k]$
Scaling	$x[rn] \;\; r \in \mathcal{Z}$	$\longleftrightarrow$	See Section 3
Time Difference	$x[n] - x[n-1]$	$\longleftrightarrow$	$\left(1 - e^{-\mathrm{i}\frac{2\pi}{N}k}\right) X[k]$
Summation	$\sum_{m=-\infty}^{n} x[m]$	$\longleftrightarrow$	$\left(1 - e^{-\mathrm{i}\frac{2\pi}{N}k}\right)^{-1} X[k]$ if $X[0] = 0$
Real $\sim$ HS	$\mathrm{RE}\{x[n]\}$	$\longleftrightarrow$	$\mathbb{HS}\{X[k]\}$
Imaginary $\sim$ HAS	$\mathrm{IM}\{x[n]\}$	$\longleftrightarrow$	$-\mathrm{i}\mathbb{HAS}\{X[k]\}$
HS $\sim$ Real	$\mathbb{HS}\{x[n]\}$	$\longleftrightarrow$	$\mathrm{RE}\{X[k]\}$
HAS $\sim$ Imaginary	$\mathbb{HAS}\{x[n]\}$	$\longleftrightarrow$	$\mathrm{i}\mathrm{IM}\{X[k]\}$
Duality	$X[n]$	$\longleftrightarrow$	$Nx[-k]$

Discrete Fourier Transform Pairs

No.	**Periodic (N) Signal:** $x[n]$	**Periodic (N) Transform:** $X[k]$
$\langle 1 \rangle$	1	$N \sum_{m=-\infty}^{\infty} \delta_K[k - mN]$
$\langle 2 \rangle$	$\sum_{m=-\infty}^{\infty} \delta_K[n - mN]$	$1 \ \forall\, k$
$\langle 3 \rangle$	$e^{\mathrm{i}\frac{2\pi}{N}k_0 n}$	$N \sum_{m=-\infty}^{\infty} \delta_K[k - k_0 - mN]$
$\langle 4 \rangle$	$\sum_{m=-\infty}^{\infty} \delta_K[n - n_0 - mN]$	$e^{\mathrm{i}\frac{2\pi}{N}kn_0}$
$\langle 5 \rangle$	$\cos\left(\frac{2\pi}{N}k_0 n\right)$	$\frac{N}{2} \sum_{m=-\infty}^{\infty} (\delta_K[k - k_0 - m] + \delta_K[k + k_0 - m])$
$\langle 6 \rangle$	$\sin\left(\frac{2\pi}{N}k_0 n\right)$	$\frac{N}{2\mathrm{i}} \sum_{m=-\infty}^{\infty} (\delta_K[k - k_0 - m] - \delta_K[k + k_0 - m])$
$\langle 7 \rangle$	$\sum_{m=-\infty}^{\infty} \mathrm{Drect}_{N_0}[n - mN] \quad 0 < N_0 < N/2$	$(2N_0 + 1)\mathrm{dinc}_{N_0}\left(\frac{k}{N}\right)$
$\langle 8 \rangle$	$\sum_{m=-\infty}^{\infty} \mathrm{Dtrian}_{N_0}[n - mN] \quad 0 < N_0 < N/4$	$\left((2N_0 + 1)\mathrm{dinc}_{N_0}\left(\frac{k}{N}\right)\right)^2$

23.2 Relation Between Fourier Family Members

The development in this section is intended to complement the theoretical view of Fourier Family presented in Section 22.2 by considering the real world signals which can be represented using each transform. The FS, DTFT and DFT are shown to be special cases of the Fourier Transform. These special cases allow us to maintain the relevant information in the signal with less overhead than the FT. An understanding of the material in this section is also required to implement a simulation of a continuous time spectral processing system using a digital computer (or implementation in DSP hardware for that matter).

We know that any square integrable function can be represented by the FT. But this includes time-limited signals which we know can be represented using the FS. Similarly we will see that band-limited (frequency-limited) signals can be represented by the DTFT. The DFT will also be shown to be the special case corresponding to a signal which is both time and band limited (approximately).

23.2.1 Band Limited Signals - DTFT and Time Sampling

If we start with a signal, $x(t)$, which is bandlimited, that is $X(f) = 0 \quad \forall\ |f| > B$, then we can completely represent the signal by a countable number of time samples. Mathematically, we

Definition of the Z-transform

$$\mathscr{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$Z\{x(t)\}$ or $Z\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t)+bx_2(t)$	$aX_1(z)+bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z)-zx(0)$
4.	$x(t+2T)$	$z^2X(z)-z^2x(0)-zx(T)$
5.	$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
6.	$x(t+kT)$	$z^kX(z)-z^kx(0)-z^{k-1}x(T)-\ldots-zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z)-z^kx(0)-z^{k-1}x(1)-\ldots-zx(k1-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz\frac{d}{dz}X(z)$
11.	$kx(k)$	$-z\frac{d}{dz}X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^{a})$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z\frac{d}{dz}X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z\to\infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$ if $(1-z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k)=x(k)-x(k-1)$	$(1-z^{-1})X(z)$
19.	$\Delta x(k)=x(k+1)-x(k)$	$(z-1)X(z)-zx(0)$
20.	$\sum_{k=0}^{n} x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a}x(t,a)$	$\frac{\partial}{\partial a}X(z,a)$
22.	$k^mx(k)$	$\left(-z\frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^{n} x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ $1 \quad k=0$ $0 \quad k\neq 0$	1
2.	–	–	$\delta_0(n-k)$ $1 \quad n=k$ $0 \quad n\neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^k $k=1,2,3,\ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$